FAR BEYOND

MAT122

Quadratic Function

Stony Brook University

Quadratics - Intro

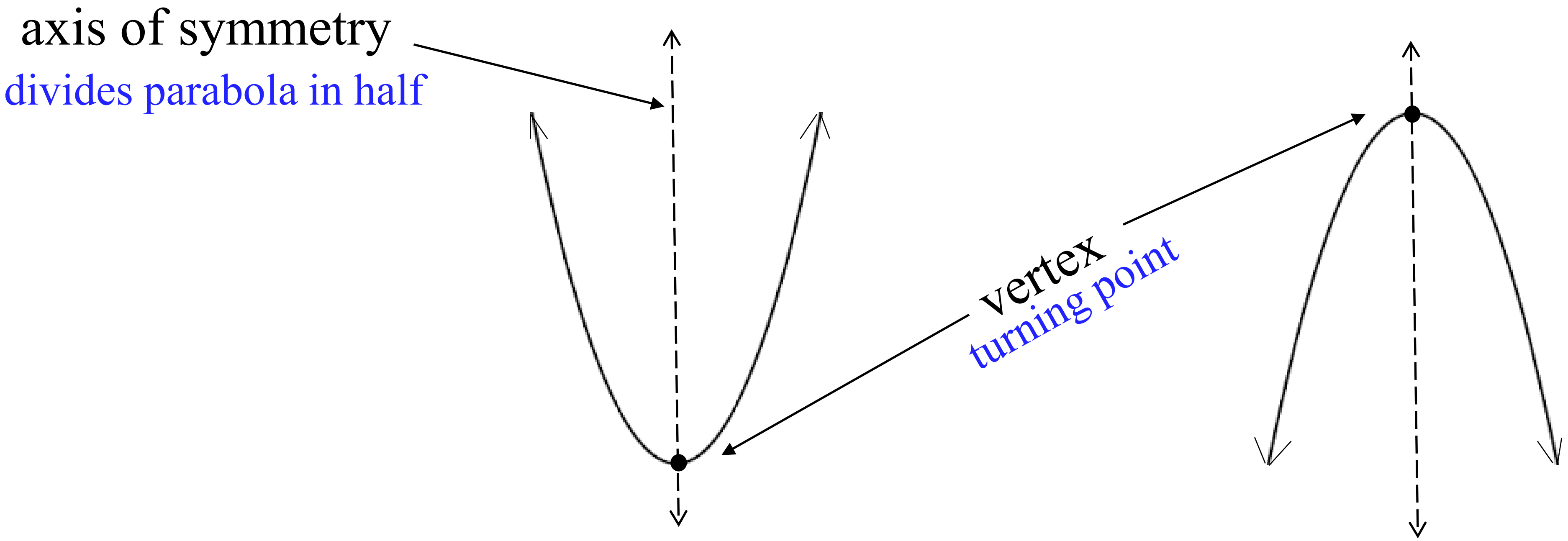
The quadratic function graphs as a parabola.

$$\boxed{\begin{array}{c} f(x) = ax^2 + bx + c \\ \text{where } a \neq 0 \end{array}}$$

a called “leading coefficient”

when $a > 0$
parabola opens upward

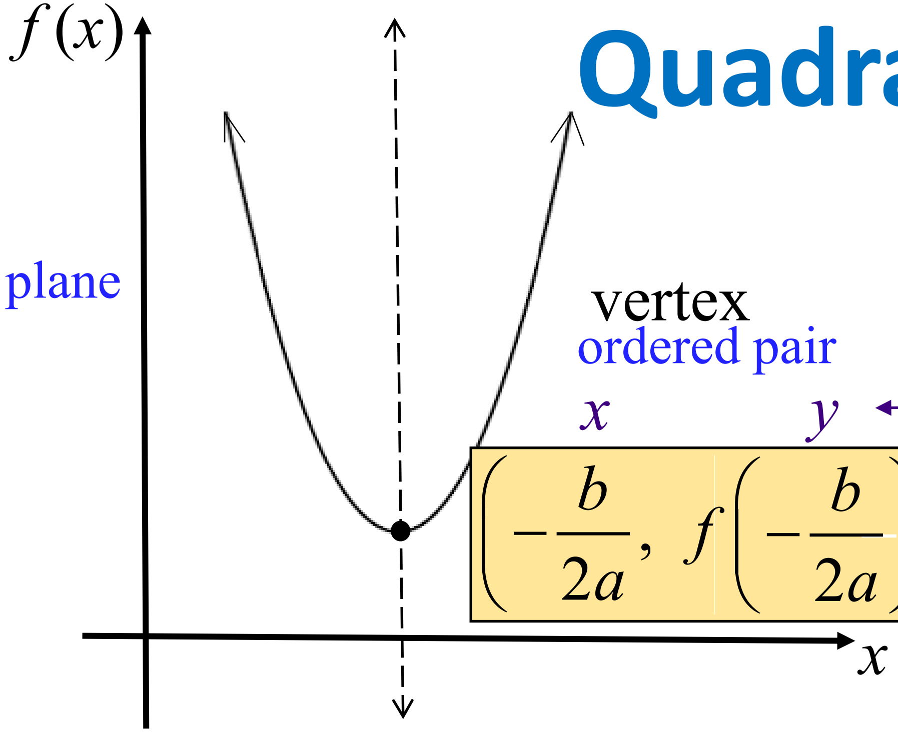
when $a < 0$
parabola opens downward



Quadratics – Open Up

$$f(x) = ax^2 + bx + c$$
where $a \neq 0$

put on plane



vertex
ordered pair

x $\quad$ y ← evaluate at $x = -\dfrac{b}{2a}$

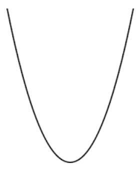
$$\boxed{\left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)}$$

axis of symmetry: $\boxed{x = -\dfrac{b}{2a}}$

vertical line -
write as an equation!

$a > 0$

ex. Graph $f(x) = x^2 + 6x + 10.$ $\quad$ expect



axis of symmetry: $x = -\dfrac{b}{2a} \rightarrow x = -\dfrac{6}{2(1)} = -3$

vertex y-value: $y = f\left(-\dfrac{b}{2a}\right) = f\left(-3\right) = (-3)^2 + 6(-3) + 10$

x-value

$= 9 - 18 + 10$

$= -9 + 10 = 1$

vertex: $(-3,1)$

y-intercept (there will be exactly 1): $f(0) = 10$ $\qquad$ $(0,10)$

set $x = 0$

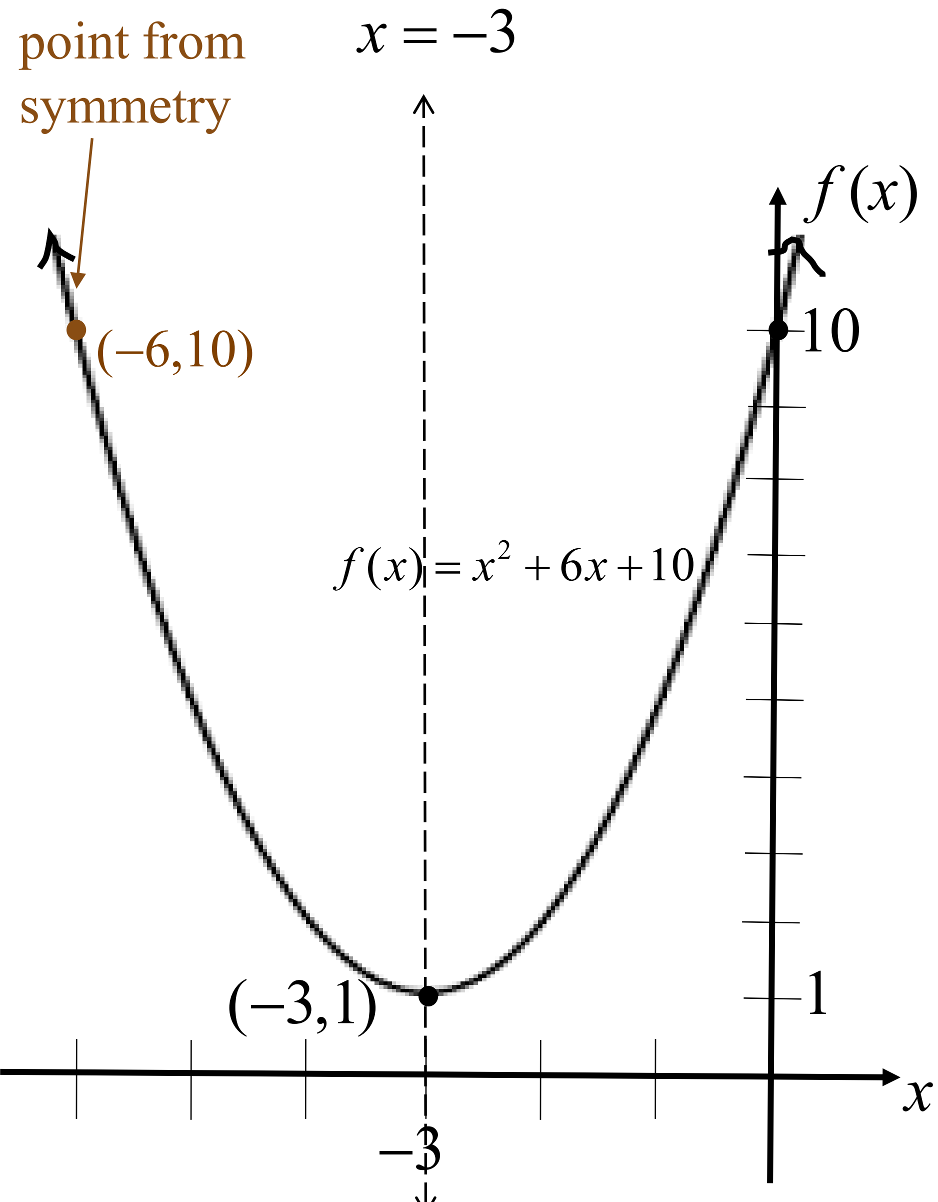
write intercepts as ordered pairs

(expect 0-2 x-intercepts) ⟶ x-intercept(s): NONE $\quad$ will show why later

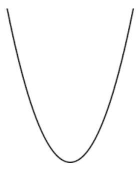
set $y = 0$

Quadratics – Open Up (cont'd)

$$f(x) = ax^2 + bx + c$$
where $a \neq 0$



$a > 0$

ex. Graph $f(x) = x^2 + 6x + 10.$ expect 

axis of symmetry: $x = -\dfrac{b}{2a} \rightarrow x = -\dfrac{6}{2(1)} = -3$

vertex y-value: $y = f\left(-\dfrac{b}{2a}\right) = f(-3) = (-3)^2 + 6(-3) + 10$

x-value

$= 9 - 18 + 10$

$= -9 + 10 = 1$

vertex: $(-3,1)$

y-intercept: $f(0) = 10 \Rightarrow (0,10)$

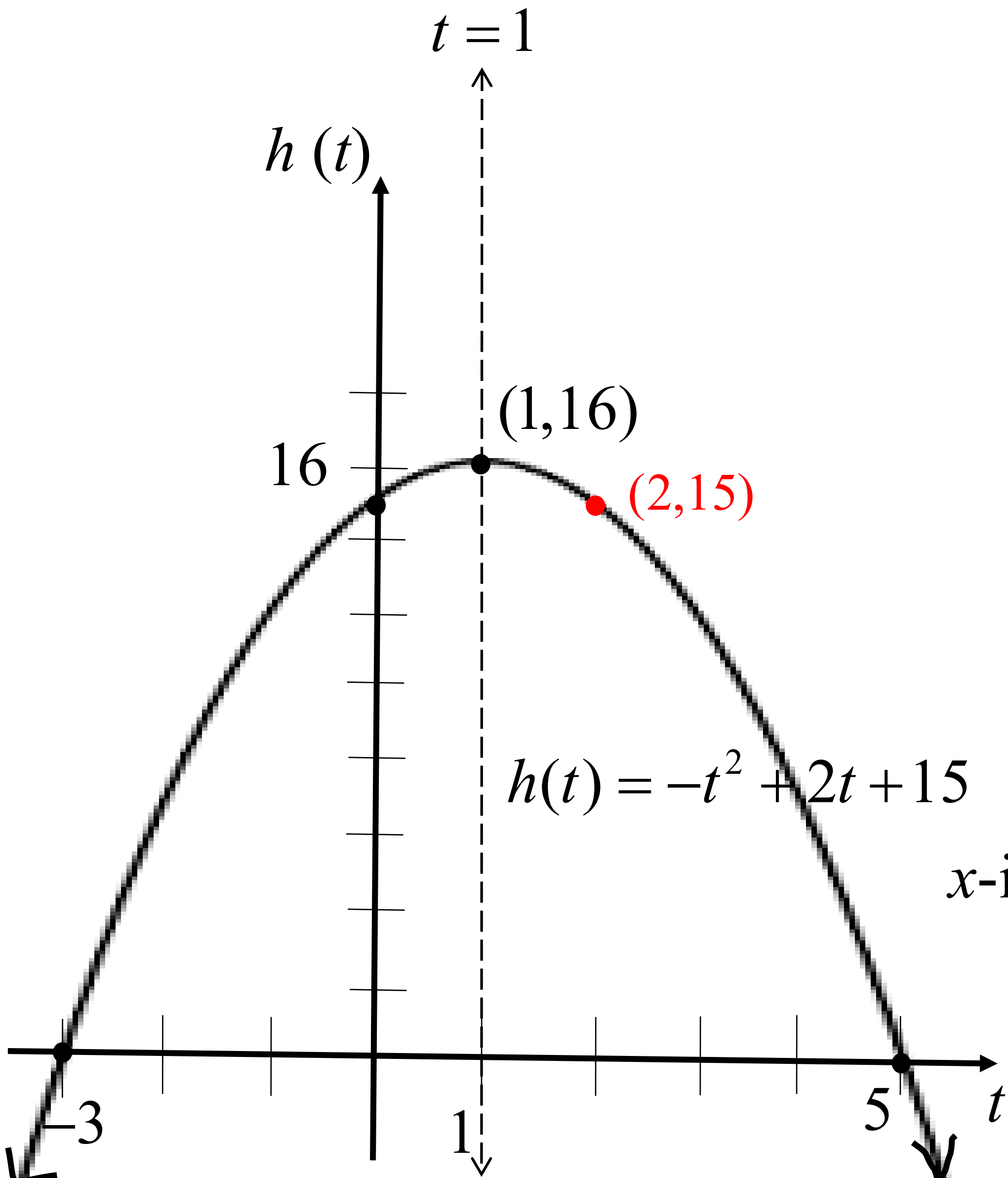
set $x = 0$

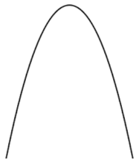
x-intercept: NONE

set $y = 0$

Quadratics –Open Down

$$f(x) = ax^2 + bx + c$$
where $a \neq 0$



expect  $a < 0$

ex. Graph $h(t) = -t^2 + 2t + 15.$

axis of symmetry: $t = -\dfrac{b}{2a} \to t = -\dfrac{2}{2(-1)} = 1$

vertex y-value: $y = h\left(-\dfrac{b}{2a}\right) = h(1) = -(1)^2 + 2(1) + 15$
$= 16$ Do: simplify

vertex: $(1,16)$

y-intercept: $h(0) = \overset{c}{15} \Rightarrow (0,15)$

x-intercept(s): $-t^2 + 2t + 15 = 0$ multiply by $-$
$t^2 - 2t - 15 = 0$ factor
$(t+3)(t-5) = 0$
$t + 3 = 0 \quad t - 5 = 0$
$t = -3 \quad t = 5 \Rightarrow (-3,0),\ (5,0)$

Quadratic Formula

Sometimes a quadratic does not factor into integers or simple fractions.
Quadratic Formula will ALWAYS work to find x-intercept(s), if any.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now revisit $f(x) = x^2 + 6x + 10$.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(10)}}{2}$$

$$= \frac{-6 \pm \sqrt{36 - 40}}{2}$$

$$= \frac{-6 \pm \sqrt{-4}}{2}$$

negative radical

$\therefore$ no REAL x-intercepts

Revisit x-intercept on $h(t) = -t^2 + 2t + 15$.

$a = -1$ $b = 2$ $c = 15$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(-1)(15)}}{2(-1)}$$

$$= \frac{-2 \pm \sqrt{4 + 4(15)}}{-2}$$

$$= \frac{-2 \pm \sqrt{64}}{-2}$$

$$= \frac{-2 \pm 8}{-2}$$

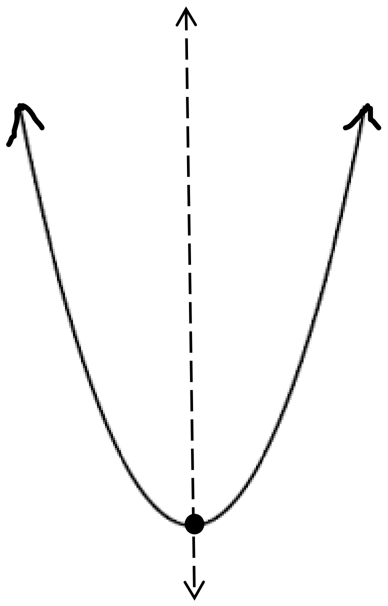
$$t_1 = \frac{-2+8}{-2} = \frac{6}{-2} = \boxed{-3}$$

$$t_2 = \frac{-2-8}{-2} = \frac{-10}{-2} = \boxed{5}$$

Application of the Vertex

Recall the sign of the leading coefficient, a, determines if parabola opens up or down.

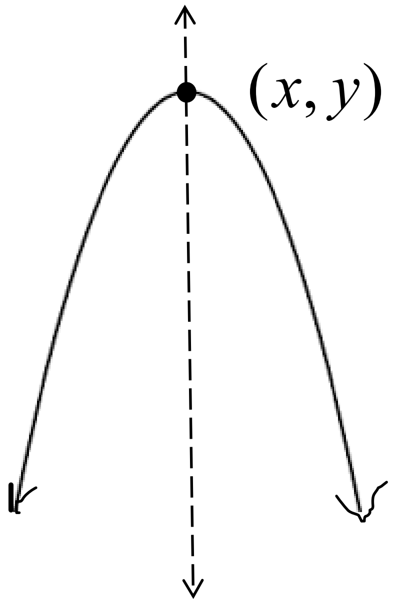
when $a > 0$



vertex is a minimum

when $a < 0$

vertex is a maximum



time

(x, y) or $(t, h(t))$

height

time "x" is <u>when</u> maximum occurs
height "y" is <u>value</u> of maximum

or "maximum value"